

Chapter 4 — Fractions, Decimals, and Percents as Three Languages — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–65 minutes each (see pacing table below)
Materials	Fraction strips, 100-square grids, place-value charts, number lines from 0 to 2, cards labeled with equivalent forms, calculators for checking, the four chapter diagrams
Launch question	“Maya writes $\frac{3}{4}$, Leo writes 0.75, Aisha writes 75%. Are they describing different amounts, or the same amount?”
Key representation	The shared number line and hundred grid — keep one visible throughout instruction
Anticipated responses	Correct: names the value as constant across forms and locates it on a shared number line. Partial: converts correctly but can’t explain why the forms agree. Incorrect: “0.6 = 6%” or similar tenths/hundredths confusion.
If the readiness check fails	Reteach decimal place value and percent-as-per-100 from Chapter 2 before starting the Launch; do not skip the concrete grid-matching step
Talk prompt	“Does this conversion change the value, or only the form?”
Exit ticket	4 items — convert fraction to decimal/percent, convert percent to fraction/decimal, explain the tenths/hundredths distinction, state a check method (see Section 14)
Reteach / extend	Reteach: benchmark values only ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, 1), hundred grid and strip. Extend: repeating decimals, values beyond 1, multiple conversion routes.

Purpose and pacing

Chapter 4 should make equivalence visible before conversion becomes procedural. Students need to experience that a fraction, decimal, and percent can occupy the same location and describe the same whole. The chapter is not a list of movement rules for decimal points.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, matching forms, and concrete grid/strip models	45–60 minutes
Lesson 2	Shared number line, conversion routes, and benchmark choices	45–60 minutes
Lesson 3	Worked examples, misconception repair, and Worksheet A	45–55 minutes
Lesson 4	Worksheets B and C, Tripwire, Exit Ticket, and targeted reteaching	50–65 minutes

Materials

Prepare fraction strips, 100-square grids, place-value charts, number lines from 0 to 2, cards labeled with equivalent forms, calculators for checking division, and the chapter diagrams. The card set should include equivalent and near-miss values such as 0.6, 6%, 60%, , 0.06, and 600%.

Teacher launch guidance

Show one shaded strip and ask three students to label it. Do not immediately confirm the labels. Ask the class to decide whether the labels point to the same amount. The goal is to establish value preservation as the standard for conversion.

When students use a decimal-point shortcut, ask what operation the shortcut represents. “Move two places” should be connected to multiplying by 100 or dividing by 100. If students cannot explain the operation, return to the hundred grid.

Teacher prompts

Purpose	Prompt
Preserve the whole	“What does 100% represent here?”
Use place value	“How many hundredths are in this decimal?”
Connect forms	“Where would each form sit on the same number line?”
Choose a route	“Is there a benchmark that makes this conversion easier?”
Check magnitude	“Is the value near 0, , 1, or beyond 1?”
Diagnose an error	“What did the student count: tenths, hundredths, or thousandths?”
Communicate	“Which form is most useful for this situation, and why?”

Differentiation

Students needing concrete support

Start with $\frac{1}{2}$, $\frac{1}{4}$, and 1. Use strips and 100-square grids before less familiar fractions. Keep the same value visible in all three forms. Have students point to the same location before writing symbols.

Students needing language support

Use these sentence frames:

- “The whole is _____.”
- “The fraction means _____ equal parts.”
- “The decimal has _____ tenths/hundredths/thousandths.”
- “The percent means _____ out of 100.”
- “The forms are equivalent because _____.”
- “The most efficient route is _____ because _____.”

Students ready to extend

Include repeating decimals, fractions such as $\frac{1}{3}$, values greater than 1, multiple conversion routes, and conversion-error creation. Ask students to explain why a decimal terminates when the simplified denominator has only factors of 2 and 5, while keeping the focus on representation and meaning.

Formative assessment

Record whether students can match equivalent forms, locate them on a shared number line, preserve the value while converting, choose an efficient route, and use a benchmark or representation to check. A student who performs the decimal movement correctly but cannot explain percent as per 100 should be recorded as developing rather than secure.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Convert $\frac{1}{2}$ to a decimal and a percent. Order 0.4, $\frac{1}{2}$, and 42% from least to greatest. Explain why 0.6 is 60%, not 6%. Choose the most useful form for describing $\frac{1}{2}$ of a class.

Strategy Studio guidance: Expected ideas: $\frac{1}{2}$; emphasize that percent is a per-100 notation and that values greater than 1 are legitimate. The form-choice task is about communication, not one universally correct format.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: Altered: the best form depends on communication context. Non-routine: 0.375, $\frac{3}{8}$, and 37.5% represent the same point on a number line. Repair: $0.4 = 40\%$, because percent means per 100 and $\frac{2}{5}$.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Chapter 4 Answer Key

Ready-to-Learn Check

- 0.5.
- 50%.
- 25 out of 100 or one-fourth.
- $\frac{1}{2} = 0.5$, so 0.5 is greater.
- .
- Three-fourths of the distance from 0 to 1.
- Per hundred.

Worksheet A

- 0.5; 50%.** Likely wrong answer: 5% (writes the decimal digits as a percent without multiplying by 100). Misconception: tenths confused with hundredths — the chapter's central error. Next move: "how many hundredths are in 0.5? Shade that many squares on a hundred grid."
- 0.25; 25%.** Likely wrong answer: 2.5% (same digit-transcription error, one place off). Misconception: not multiplying the decimal by 100 to reach the percent. Next move: "0.25 means 25 hundredths — what percent is that?"
- 0.75; 75%.** Likely wrong answer: 7.5% (same recurring error). Misconception: tenths/hundredths confusion. Next move: shade 75 squares on a hundred grid and count them as a percent.
- 0.2; ; 20%.** Likely wrong answer: 2% for the percent (same digit-

transcription error). Misconception: treating the decimal's digits as the percent directly. Next move: "0.2 is how many tenths? How many hundredths is that?"

- 0.5 = 50%; 25% = ; = 0.75; 0.75 = 75%; = 25%; 50% = .** Likely wrong answer: pairs 25% with instead of (swaps the two most common benchmark pairs). Misconception: memorizing benchmark pairs without checking the actual value. Next move: "does 25% mean one of four equal parts, or three of four?"
- 25 squares shaded.** Likely wrong answer: 4 squares (treats "25%" as if the 25 itself were a small count, or confuses it with 's denominator). Misconception: not connecting 25% to "25 out of 100." Next move: "25% means 25 out of how many? How many squares is that?"
- $< 0.3 < 35\%$ because $0.25 < 0.30 < 0.35$.** Likely wrong answer: orders 0.3 as the smallest, without converting to 0.25 first, since "" doesn't visually look like a decimal near 0.3. Misconception: comparing forms without converting to a common form first. Next move: "convert all three to decimals before comparing."
- Both are three of four equal parts, or 75 of 100.** Common flaw: states they're "both 75" without explaining what makes them equal

(the shared equal-parts meaning).
Next move: ask the student to show both on the same hundred grid.

9. **= 1.25, located beyond 1.** Likely wrong answer: places the point between 0 and 1, most often near 0.54, misreading the fraction as a decimal-like value less than 1. Misconception: not recognizing that a numerator larger than the denominator means the value exceeds 1. Next move: "is 5 more or less than 4? What does that tell you about the size of the fraction?"
 10. **The fraction shows equal parts most directly, although all three forms can be correct.** Common flaw: picks one form as "the right answer" instead of explaining that the best form depends on purpose. Next move: ask what each form is good for.
 11. **Closer to 0; 8%.** Likely wrong answer: 80% — the chapter's signature error, confusing eight-hundredths with eighty-hundredths. Misconception: tenths/hundredths confusion. Next move: "how many hundredths are in 0.08? Compare it to 0.80 on a number line."
 12. **Use a number line, benchmark, hundred grid, inverse conversion, or equivalent fraction.** Common flaw: names "redo the same steps" as the check, which doesn't independently verify the answer. Next move: ask for a check method that uses a different representation than the one used to solve.
- "does 3 fifths mean 3 divided by 5, or 5 divided by 3?"
 2. **0.875; 87.5%.** Likely wrong answer: 87% (drops the .5, rounding without noticing). Misconception: losing precision by rounding an exact decimal. Next move: "does $7 \div 8$ divide evenly? Carry the division one more place."
 3. **0.45; 45%.** Likely wrong answer: 2.22 (computes $20 \div 9$ instead of $9 \div 20$). Misconception: reversing the division, the same error as #1. Next move: "which number is being divided, and by what?"
 4. **; 62.5%.** Likely wrong answer: left unsimplified, or 6.25% (misplaces the decimal when converting to percent). Misconception: not simplifying the fraction, or a place-value slip in the percent conversion. Next move: "what do 625 and 1000 share as a common factor? And does 0.625 mean 62.5 hundredths or 6.25 hundredths?"
 5. **; 8%.** Likely wrong answer: 80% — the recurring tenths/hundredths error. Misconception: same central confusion as Worksheet A #11. Next move: "how many hundredths are in 0.08?"
 6. **; 65%.** Likely wrong answer: left unsimplified. Misconception: stopping before simplifying to lowest terms. Next move: "what number divides evenly into both 65 and 100?"
 7. **0.45; .** Likely wrong answer: 4.5 for the decimal (drops a zero, misplacing the decimal point). Misconception: not dividing by 100 correctly when converting percent to decimal. Next move: "45% means 45 out of 100 — write that as hundredths."

Worksheet B

1. **0.6; 60%.** Likely wrong answer: 1.67 (computes $5 \div 3$ instead of $3 \div 5$). Misconception: reversing the dividend and divisor in the fraction-to-decimal division. Next move:

8. **1.2; .** Likely wrong answer: 0.12 for the decimal, or rejects the problem as impossible since it's "over 100%." Misconception: percent restricted to values between 0 and 100. Next move: "what does 100% represent? What would 120% mean?"
9. **0.7 is 70%, so it is greater than 7%.** Likely wrong answer: says they are equal, or that 7% is greater, by comparing the digit "7" in both without accounting for place value. Misconception: comparing forms without converting to the same form first. Next move: "convert both to percents — what is 0.7 as a percent?"
10. **0.08 is eight hundredths, while 80% is eighty hundredths, or 0.80.** Common flaw: states the conclusion without showing the hundredths count for both values. Next move: ask the student to shade both amounts on separate hundred grids.
11. **Benchmark: 25% is one-fourth of 64, which is 16; decimal: $0.25 \times 64 = 16$.** Likely wrong answer: in the decimal route, computes 0.25×64 incorrectly (for example, 160, misplacing a decimal point). Misconception: arithmetic slip in the multiplication, not a conceptual error. Next move: estimate first — $64 \div 4$ should be close to the answer.
12. **Answers vary; for example, $1.25 = 125\%$.** Common flaw: gives a value less than 1 by mistake, forgetting the requirement that it be greater than 1. Next move: check that the fraction's numerator exceeds its denominator.
- without multiplying by 100). Misconception: treating "7 out of 10" as if it were already "7 out of 100." Next move: "7 tenths equals how many hundredths?"
2. **25% discount is of the original price; decimal 0.25.** Common flaw: gives the fraction or decimal but not both, or states the discount amount instead of the fraction/decimal describing it. Next move: ask for all three forms of "one-fourth off."
3. **0.45 = 45%, so 0.45 is greater than 44%.** Likely wrong answer: says 44% is greater, from comparing "0.45" and "44" as if they were on the same scale without converting both to the same form first. Misconception: comparing across forms without a shared reference. Next move: "convert both to percents, or both to decimals, before comparing."
4. **125% means 1.25 times the original height, or one whole plus one-fourth.** Common flaw: says the plant "grew by 125%" (confusing the growth amount with the new total). Next move: "does 125% describe the new height, or how much it grew?"
5. **= 0.375, exactly; 0.38 is a rounded hundredths approximation.** Likely wrong answer: accepts 0.38 as exact. Misconception: not recognizing that rounding to hundredths loses information for a thousandths-place value. Next move: "does $3 \div 8$ divide evenly, or does it need a third decimal place?"
6. **0.4 = 40%; the student confused tenths and hundredths.** Likely wrong answer: agrees that $0.4 = 4\%$, the exact error given in the problem. Misconception: the

Worksheet C

1. **0.7; 70%.** Likely wrong answer: 7% (keeps the numerator as the percent

chapter's central tenths/hundredths confusion. Next move: "0.4 is how many tenths? How many hundredths is that the same as?"

7. **$3 \div 8 = 0.375$, then $0.375 \times 100 = 37.5\%$; a benchmark or fraction-over-100 route is also acceptable.** Common flaw: finds only one route and can't produce or compare a second. Next move: ask for a route that doesn't start with division.
8. **Answers vary; 250% can describe a quantity that is 2.5 times the original.** Common flaw: creates a context where the described quantity is actually less than or equal to the original, which contradicts a percent greater than 100. Next move: check that the situation genuinely involves more than one whole.
9. **A decimal display may be useful for measurement, but the equivalent fraction should be retained as evidence of the amount.** Common flaw: picks one form as universally "correct" rather than discussing the trade-off between measurement convenience and equal-parts meaning. Next move: ask what information the fraction preserves that the decimal display might obscure.
10. **8%; the value is close to 0, not to 100%.** Likely wrong answer: 80%, the recurring tenths/hundredths error. Misconception: same central confusion as items 1 and 6. Next move: shade 0.08 and 0.80 on separate hundred grids and compare.
11. **$0.65 = 65\%$. The number of people (250), the number of days (3), and the number of volunteers (4) were not needed**

to find the percent. Likely wrong answer: uses 250 in the calculation somehow (for example, finding "0.65 of 250 people"), even though the question only asks for the percent form of 0.65. Misconception: assuming every number in a word problem must be used. Next move: "what does the question actually ask for? Which number answers that?"

12. **Answers vary; must describe a situation where , 0.4, and 40% all describe the same amount, with the meaning of the value stated in context.** Common flaw: gives a context but only names one form, or the three forms don't actually describe the same value. Next move: ask the student to show all three forms agree on the same hundred grid or number line.

Extend Your Thinking

1. 0.5 is five tenths, or fifty hundredths; 0.05 is five hundredths.
2. Yes; $= = 0.5$.
3. 125% means 125 parts per 100, so it is greater than one whole.
4. $= 0.4375 = 43.75\%$; division followed by multiplication by 100 is a general route, while a fraction-over-100 route can also be used.
5. Answers vary; the diagnosis must identify the place-value or per-100 error.

Tripwire and Exit Ticket

Tripwire: $0.08 = 8\%$, not 80%.

Exit Ticket: $= 0.625 = 62.5\%$;

$37.5\% = 0.375 =$;

0.6 is 60 hundredths;

checks may use a hundred grid, number line, benchmark, or equivalent representation.

